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SHORTER COMMUNICATION

NOTE ON THE EFFECT OF THERMAL DISTORTION ON CONSTRICTION RESISTANCE

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NOMENCLATURE

A_i, area of contact of a single contact spot;
a_i, radius of a single area of contact;
A, fraction of actual contact;
B, H, abbreviations;
C_k, $\alpha_k(1+\nu_k)/K_k$;
E_k, Young's modulus;
E', $\sum_{k=1,2}(1-\nu_k^2)/E_k$;
F_i, force due to a single contact spot;
h_i, compliance;
K_k, thermal conductivity;
K_0, $2K_1K_2/(K_1+K_2)$;
N, number of asperities per unit area A_a;
P, total contact force per unit area A_a;
p, pressure;
Q, total heat flux per unit area A_a;
Q_i, heat flux through a single contact area A_i;
R_i, radii of curvature of the contacting asperities when unheated;
R, $R_1R_2/(R_1+R_2)$;
r, r', variable radii;
T_k, temperatures;
ΔT, temperature difference;
u, separation of the mean planes;
z_k, axial coordinates.

Greek symbols

α_k, coefficient of thermal expansion;
δ, $B^*/(2\sqrt{R^*})$;
λ, constriction resistance;
ν, Poisson's ratios;
ζ, integration variable;
σ, RMS roughness of the surface;
φ, thermal contact conductance;
ψ, contact resistance factor.

Superscripts

*, denotes normalization with respect to σ.

Subscripts

0, interface or mean value;
i, single contact spot;
k, $k = 1, 2$ solids 1,2.

INTRODUCTION

DUE TO the inevitable roughness of all surfaces in nature the heat flow through the interface between two apparently conforming solids is less than that which would be obtained in the presence of perfectly smooth bodies. The actual contact is restricted to a few randomly distributed small areas within the apparent contact area. The distortion of the heat flow causes a change in thermal contact resistance or constriction resistance, which depends on the direction of heat flow.

In a paper [1] Barber gives a deterministic mathematical treatment of the effect of thermal distortion on the thermal contact resistance between two semi-infinite solids of different materials. The present work presents an attempt to clear up the influence of the thermal distortion due to heat flow in the case of rough nominally flat surfaces in perfect thermal contact throughout the final circular contact areas A_i of radius a_i of the asperities in a vacuum under conditions of negligible radiation. Both the asperities and the substrate are assumed to behave elastically. For the sake of simplicity and to have a first impression of the thermal-effect an exponential probability distribution of asperity heights is assumed.

ANALYSIS

The undistorted surfaces of the solids are spheres, whereas the distorted surfaces are approximated by paraboloids

$$z_k = r^2\left\{\frac{1}{2R_k}+\frac{C_k}{\pi a_i}(T_1-T_2)K_0(1-\ln 2)\right\} \quad (k = 1,2), \quad (1)$$

where R_k are the radii of curvature, T_k the "temperatures at infinity" of the solids, $K_0 = 2K_1K_2/(K_1+K_2)$ with K_k the conductivities and C_k thermoelastic constants [1] (see Table 1).

Table 1. Thermoelastic constants of some materials (cgs, cal, °C—System, $\nu_k = 0{\cdot}3$)

Substance	Density ρ	Spec. heat c	Diff. coeff. κ	Lin. exp. coeff. $\alpha \times 10^6$	Conduct. coeff. K	$C = \frac{(1+\nu_k)}{K_k}$ $C \times 10^6$
Copper	8·94	0·0914	1·14	16·2	0·93	22·64
Silver	10·49	0·0556	1·71	19·7	1·00	25·60
Aluminium	2·70	0·2060	0·86	23·8	0·48	64·50
Bronze	8·50	0·09	0·33	18·0	0·25	93·60
Steel	7·85	0·118	0·12	11·1	0·11	131·18
Glass (flint)	2·40	0·20	0·0058	5·0	0·0028	2321·42
Concrete	2·30	0·23	0·0042		0·0022	

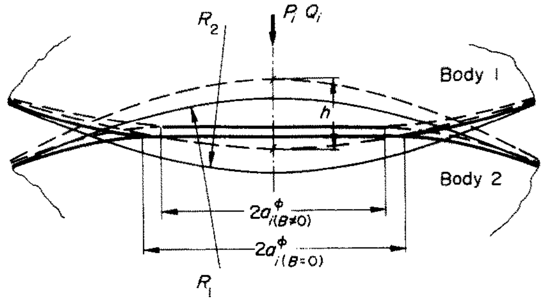

FIG. 1. Thermal contact geometry (——, nominal surfaces; ——, Hertzian deformed surfaces; ---, thermal distorted nominal surfaces; —-—, thermal distorted surfaces in contact).

The necessary pressure distribution $p(r)$ to make the surfaces (1) conform throughout the contact circle A_i is calculated by extending Hertz's analysis to the anisothermal case:

$$\frac{1}{\pi E'}\iint_{A_i}\frac{p(r')r'}{|r-r'|}\mathrm{d}r' = h_i-(z_1+z_2) \tag{2}$$

$$\iint_{A_i}\frac{r'}{|r-r'|}\sqrt{\left(1-\frac{r'^2}{a_i^2}\right)}.\mathrm{d}r' = \frac{\pi a_i^2}{2}\int_0^\infty\left[1-\frac{r^2}{a_i^2+\xi}\right]\frac{\mathrm{d}\xi}{(a_i^2+\xi)\sqrt{\xi}}.$$

Noting that

$$p(r) = 3F_i/(2\pi a_i^2)\sqrt{\left(1-\frac{r^2}{a_i^2}\right)}$$

and considering (1), a comparison of the r.h.s. in (2) leads to the formulae

$$\begin{aligned} F_i &= 4E'a_i^3/(3R)+8(C_2-C_1)(T_1-T_2)K_0(1-\ln 2)E'a_i^2/(3\pi) \\ h_i &= a_i^2/R+2(C_2-C_1)(T_1-T_2)K_0(1-\ln 2)a_i/\pi, \end{aligned} \tag{3}$$

where h_i is the compliance, F_i the total force, $R = R_1R_2/(R_1+R_2)$, v_k the Poisson's ratios, E_k the Young's moduli,

$$1/E' = \sum_{k=1,2}(1-v_k^2)/E_k.$$

This can be interpreted as follows: equation (3) gives the load F_i and the compliance h_i required to cause the solids 1, 2 to conform over a circular area of radius a_i when their extremities are maintained at temperatures T_1, T_2, respectively (Fig. 1).

The second equation in (3) permits the determination of the contact radius for a given compliance:

$$a_i = -B/2+\sqrt{[(B/2)^2+h_iR]} \tag{4}$$

where

$$B = \frac{2}{\pi}R(C_2-C_1)(T_1-T_2)K_0(1-\ln 2) \tag{5}$$

and the positive square root has to be chosen to ensure a positive value of a_i. Tables 2 and 3 show the relations between F_i, a_i and h_i for some specified values of B and demonstrate the dependence of the change of the contact area on the direction of heat flow. The cases $B = 0$ (isothermal or similar materials) reduce to the Hertzian theory.

In the following, the above solution for a single contact spot represents the basis of the conduct of rough bodies with heat flow, specifically in the contact of a rough nominally flat surface and a smooth plane.

The thermal contact conductance φ ($\lambda = \varphi^{-1}$ is the constriction resistance) is defined as [2]:

$$\varphi =: \frac{Q}{T_1-T_2} = \frac{\sum Q_i}{T_1-T_2} \tag{6}$$

Table 2. Load F_i (kP) vs $\alpha = a_i/R$ at given temperature differences (°C)

$\alpha = \dfrac{a_i}{R}$	$\Delta T = T_1-T_2$ -500	-100	0	100	500
1·00	459·95	460·71	460·91	461·10	461·87
0·75	194·90	194·34	194·45	194·55	194·98
0·50	57·37	57·56	57·61	57·66	57·85
0·40	29·34	29·47	29·50	29·53	29·65
0·30	12·36	12·43	12·45	12·46	12·53
0·25	7·14	7·19	7·20	7·21	7·26
0·20	3·65	3·68	3·687	3·695	3·725
0·15	1·534	1·551	1·555	1·560	1·577
0·10	0·451	0·459	0·461	0·463	0·471
0·05	0·055	0·057	0·0575	0·058	0·060

Table 3. Compliance h/R vs $\alpha = a/R$ at given temperature differences (°C)

$\alpha = \dfrac{a_i}{R}$	$\Delta T = T_1-T_2$ -500	-100	0	100	500
1·00	0·9979	0·9995	1·0000	1·0004	1·0020
0·75	0·5609	0·5621	0·5625	0·5628	0·5640
0·50	0·2489	0·2497	0·2500	0·2502	0·2510
0·25	0·0619	0·0623	0·0625	0·0626	0·0630
0·20	0·0395	0·0399	0·0400	0·0400	0·0404
0·15	0·0221	0·0224	0·0225	0·0225	0·0228
0·05	0·0023	0·0024	0·0025	0·0025	0·0026

where Q is the total heat flux per unit apparent area, Q_i the heat flux through a single contact spot. Let $Q_i = 2K_0a_i(T_1-T_2)/\Psi$, where $\Psi = (1-\sqrt{A})^{3/2}$ and A is the fraction of actual area of contact, then

$$\varphi = \lambda^{-1} = \frac{2K_0}{\Psi}\sum a_i. \tag{7}$$

The further development strongly depends on the kind of the summit height probability distribution. The special choice of an exponential distribution permits an analytical solution of the problem. This probability distribution is a fairly good approximation to real surfaces at low loads since it covers the higher asperities, and gives a first information on the magnitude of the thermal distortion effect.

The fraction of actual area in contact A is given by [3]:

$$A = N\pi\sigma^2R^*\exp(-u^*)[1-\sqrt{\pi}\,\delta\exp(\delta^2)\mathrm{erfc}(\delta)] \tag{8}$$

where $\delta = B^*/(2\sqrt{R^*})$, $u^* = u/\sigma$ denotes the dimensionless separation of the mean planes with respect to the RMS roughness σ of the surface, N is the number of asperities in the area A_a, $B^* = B/\sigma$ and $R^* = R/\sigma$.

The sum of the contact radii is given by

$$\sum a_i = N\sigma\int_{u^*}^\infty a_i^*\,\mathrm{e}^{-\xi}\,\mathrm{d}\xi = N\sigma\frac{\sqrt{\pi R}}{2}\mathrm{e}^{-u^*}\exp(\delta^2)\mathrm{erfc}(\delta) \tag{9}$$

and the load per unit area by [3]

$$\begin{aligned} P &= \frac{N4E'\sigma^2}{3R^*}\int_{u^*}^\infty(a_i^{*3}+B^*a_i^{*2})\mathrm{e}^{-\xi}\,\mathrm{d}\xi \\ &= \frac{N4E'\sigma^2}{3}\mathrm{e}^{-u^*}\exp(\delta^2)H(B^*) \end{aligned} \tag{10}$$

where $a_i^* = a_i/\sigma$ and

$$H(B^*) = \frac{\sqrt{R^*}}{2}\,[\delta\exp(-\delta^2)+\sqrt{\pi}(\tfrac{3}{2}-\delta^2)\mathrm{erfc}(\delta)]. \tag{11}$$

Upon eliminating u^* in the equations (9) and (10) one gets

$$\sum a_i = \frac{3\sqrt{(\pi R^*)}}{8E'} \frac{P}{\sigma} \frac{\text{erfc}(\delta)}{H(B^*)}, \tag{12}$$

and, considering the relation P/A, from (7) and (12) a transcendental relation between constriction resistance λ, total force P and total heat flux Q results by substituting $T_1 - T_2 = \lambda . Q$ in the expression for B:

$$\frac{(1-\sqrt{[A(P,Q)]})^{3/2}}{P} = K_0 \lambda \frac{3\sqrt{(\pi R^*)}}{4E'\sigma} \frac{\text{erfc}(\delta)}{H(B^*)}. \tag{13}$$

In the case of low loads, where the thermal interaction of the randomly distributed single contact spots is very weak—the adiabatic cylinder [2] becomes large—ψ tends to unity; then from (13) one gets an approximate relation between P, Q and λ of the form

$$P = \frac{\sigma}{K_0 \lambda} \frac{4E'}{3\sqrt{(\pi R^*)}} \frac{H(B^*)}{\text{erfc}(\delta)} \tag{14}$$

where B^* now is a function of the product $\lambda K_0 Q$ given in (5). Table 4 shows the relation between P, Q and λ for heat flow between stainless steel and copper specimens in contact.

Example. The thermal distortion effect due to heat flow between the interface of a stainless steel specimen, referred to as the smooth plane (index 1), and a copper specimen, referred to as the rough plane (index 2), in contact is analyzed.

Data:

Young's modulus (*St*)	$E_1 = 2{\cdot}08 \times 10^6$ kP/cm^2
Young's modulus (*Cu*)	$E_2 = 1{\cdot}25 \times 10^6$ kP/cm^2
Poisson's ratio	$\nu_1 = \nu_2 = 0{\cdot}3$
Conductivity	$K_0 = 0{\cdot}1967$ cal/(scm^3 °C)
$C_2 - C_1 = -108{\cdot}54 \times 10^{-6}$ scm^3/cal	
Asperity radius	$R = R_2 = 0{\cdot}02$ cm
Roughness	$\sigma = 5 \times 10^{-5}$ cm
Asperity density	$N = 5 \times 10^4$/cm^2
Temperatures differences	$T_1 - T_2 = (-500, -100, 0, 100, 500)$ °C

Table 4. Total load P (kP) vs constriction resistance λ (scm^2 °C/cal) at given total heat flux Q (cal/scm^2) (equation 14)

λ	Q				
	−500	−100	0	100	500
1	219·86	219·48	218·10	217·75	216·25
2	110·71	109·38	109·05	108·70	107·28
5	45·22	43·97	43·62	43·25	41·79
10	23·31	22·14	21·81	21·45	19·85

CONCLUSION

An extension of a paper by Barber on thermal distortion effects of two contacting spherical bodies is made to two rough nominally flat perfectly thermally contacting elastic solids with negligible radiation effects. The change of the radius of the real contact area of a single contact spot and of the compliance of two spherical asperities depending on the force and the magnitude and direction of the heat flow for two contacting specimens of copper and stainless steel is represented in the Tables 2 and 3 (equation 3). The theory shows, that an exponential summit height probability distribution leads to a simple relation between the total force P, the heat flux Q and the constriction resistance λ (Table 4).

In an appropriate manner, the more general case of the contact of rough spherical bodies can be solved. Distributions of summit heights other than exponential considerably increase the numerical work.

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